Stochastic approximation for chance-constrained NLPs

Rohit Kannan[1] and Jim Luedtke[1,2]

[1]Wisconsin Institute for Discovery, University of Wisconsin-Madison

[2]Department of Industrial and Systems Engineeering, University of Wisconsin-Madison

Setup

$$\nu^* = \min_{x \in X} \; f(x) \quad \text{s.t.} \; \mathbb{P}\{g(x,\xi) \leq 0\} \geq 1-\alpha$$

- X compact convex, f quasiconvex, $g : \mathbb{R}^n \times \mathbb{R}^d \to \mathbb{R}^m$
- For each $x \in X$, $g(x,\xi)$ is a continuous random variable
- **Can model** joint chance constraints, recourse structure

Applications

- Financial systems with uncertain markets
- Power grid operation under renewable energy uncertainty
- Reliable design and control under model uncertainties

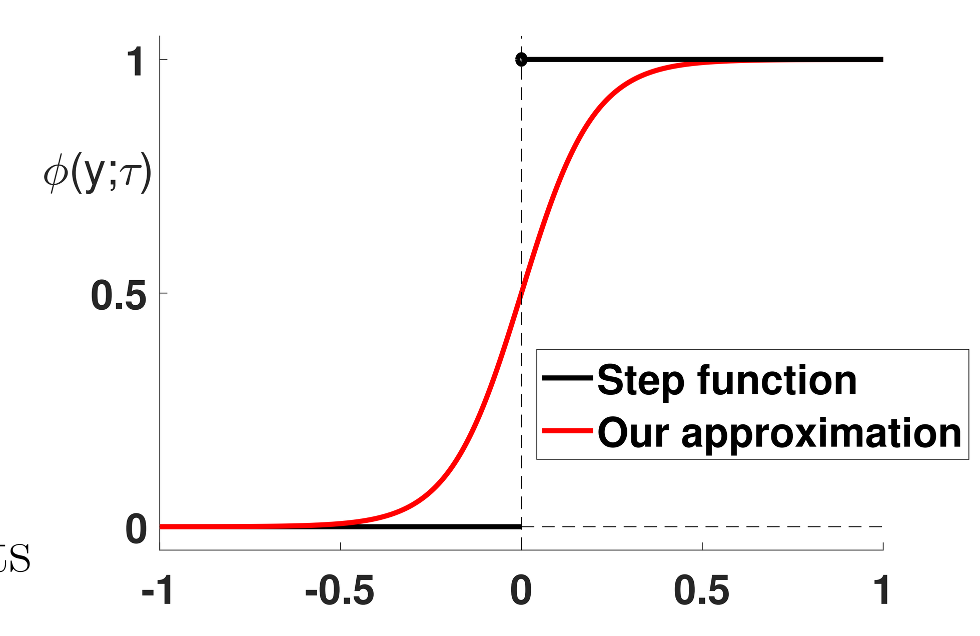
Central Hypothesis

Decision makers **want to generate efficient frontier** of optimal objective function value (ν^*) versus risk level (α), see Rengarajan and Morton (2009)

Key reformulation

$$\min_{x \in X} \; \mathbb{P}\{g(x,\xi) \not\leq 0\} \;\; \text{s.t.} \; f(x) \leq \nu \quad \equiv \quad \min_{x \in X} \; \mathbb{E}\left[\max_{j \in \{1,\cdots,m\}} [\mathbb{1}[g_j(x,\xi)]]\right] \;\; \text{s.t.} \; f(x) \leq \nu$$

- ν: specified bound
- $\mathbb{1}[\cdot]$: step function
- **Recover efficient frontier** by solving above problem
- **Challenge**: hard to get stochastic gradients



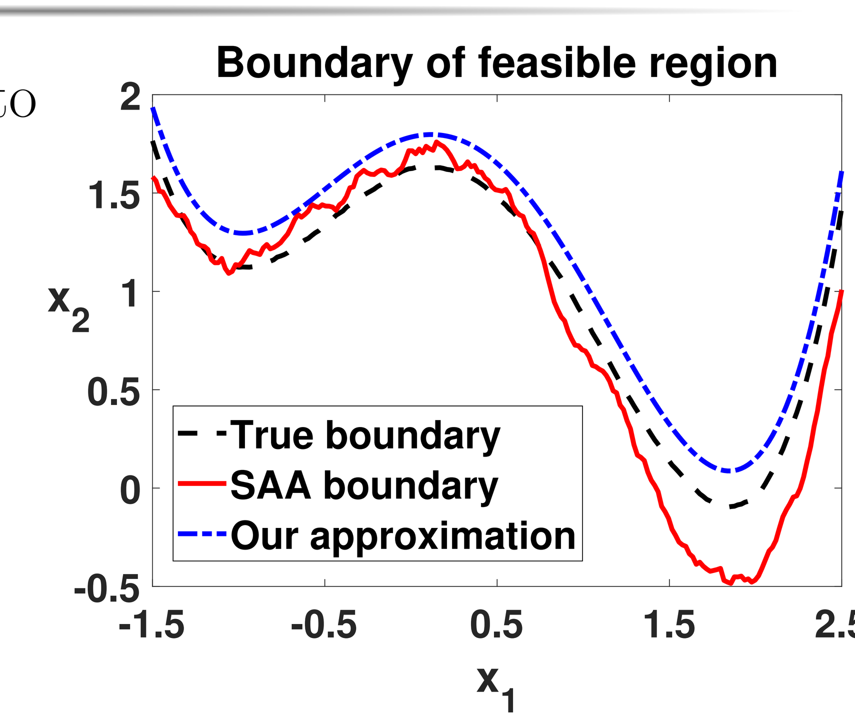
Approximating the efficient frontier

$$\min_{x \in X} \; \mathbb{E}\left[\max_j [\mathbb{1}[g_j(x,\xi)]]\right] \;\; \text{s.t.} \; f(x) \leq \nu \quad \approx \quad \min_{x \in X} \; \mathbb{E}\left[\max_j [\phi(g_j(x,\xi);\tau_j)]\right] \;\; \text{s.t.} \; f(x) \leq \nu$$

- Approximate step function $\mathbb{1}[\cdot]$ using a parametric family of smooth functions $\phi(\cdot;\tau)$
  - Tailor choice of smoothing parameter τ for each constraint
  - Related work: Tamm (1979), Norkin (1993), Geletu et al. (2017), etc.
- Solve approximation using stochastic subgradient methods
  - Related work: Norkin (1993), Lepp (1980), Andrieu et al. (2007), etc.

Motivation for stochastic approximation

- Exterior sampling may lead to **spurious local minima**, see Curtis et al. (2018)
- Solution based on exterior sampling may be suboptimal or computationally intensive to generate



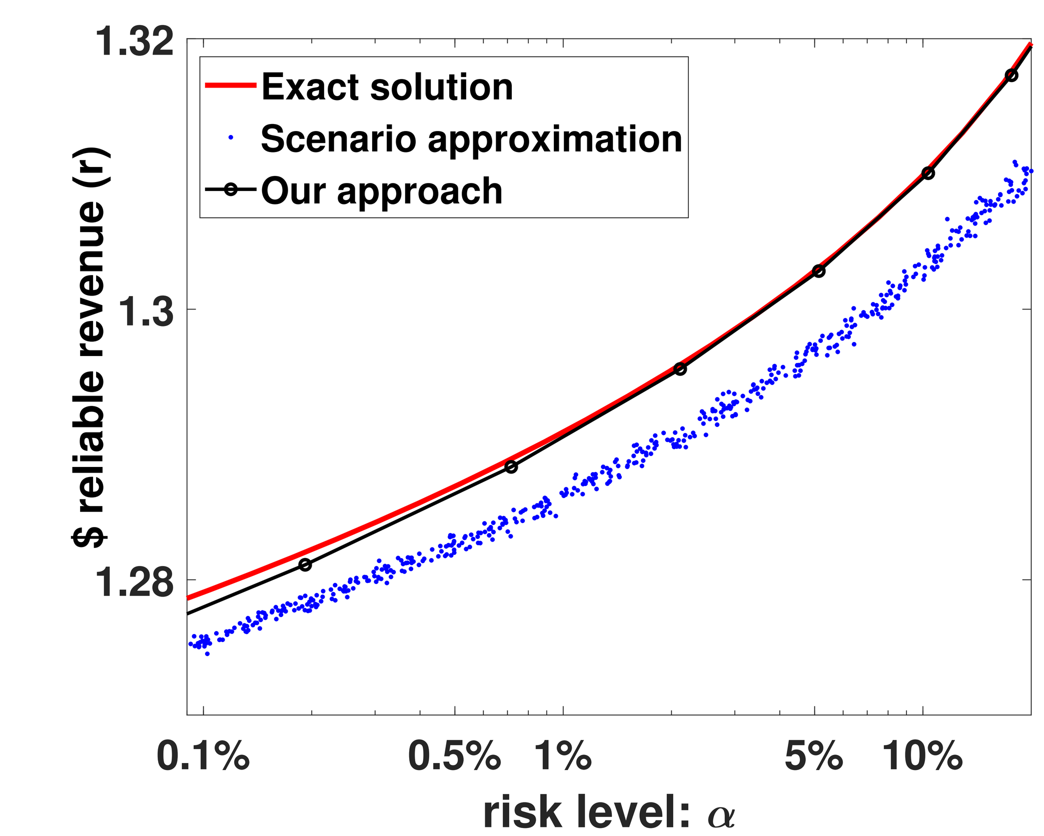
Proposed Algorithm

Input: point $\hat{x}^0 \in X$ and objective bound $\bar{\nu}^0$ obtained from scenario approximation, spacing $\tilde{\nu}$, risk lower bound α_{low}
Output: approximation $\{(\bar{\nu}^i, \bar{\alpha}^i)\}$ of the efficient frontier
repeat with $i = 1, 2, \cdots$
  Set objective bound $\bar{\nu}^i = \bar{\nu}^0 - (i-1)\tilde{\nu}$
  Determine smoothing parameters $(\tau_j)_{j=1}^m$ scaled at $\hat{x}^{i-1}$
  repeat
    Solve smooth approximation using projected stochastic subgradient algorithm to get solution $\hat{x}^i$
    Estimate risk $\bar{\alpha}^i$ of solution $\hat{x}^i$ using a random sample
    Refine approximations of step function $\left[\text{update } (\tau_j)_{j=1}^m\right]$
  until no significant decrease in $\bar{\alpha}^i$
until $\bar{\alpha}^i \leq \alpha_{low}$

Computational results

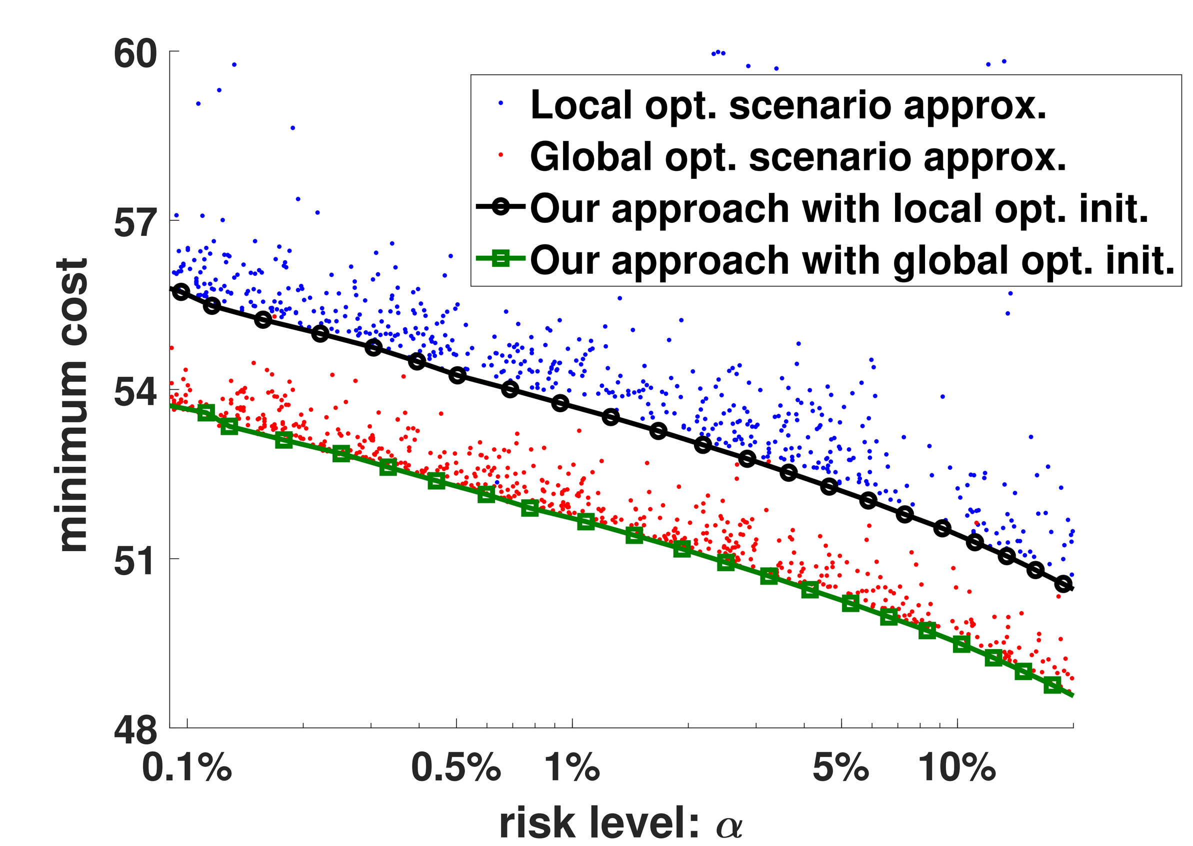
Portfolio optimization

- 1000 stocks with normally distributed returns ξ
- Budget: \$1. Invest fraction $x_i \in [0,1]$ in stock i
- Maximize reliable revenue r such that $\mathbb{P}\{\xi^T x \geq r\} \geq 1-\alpha$



Resource planning (modified from Luedtke (2014))

- Meet demands of 30 customer groups for 20 resources
- Uncertainty in resource yields and customer demands
- Chance constraints with nonconvex recourse structure



Preprint: arXiv:1812.07066. Code on GitHub.

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