Stochastic DC Optimal Power Flow with Reserve Saturation

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DC Optimal Power Flow

Notation:

- $\mathcal{V}$ = set of buses/nodes; $\mathcal{G}$ = set of generators
- $\mathcal{E}$ = set of lines/edges; $\mathcal{D}$ = set of loads
- p^0 = generator outputs; θ = phase angles

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$$p_i^{\min} \leq p_i^0 \leq p_i^{\max}, \quad \forall i \in \mathcal{V}, \quad \Big\} \text{ generation limits}$$

Other constraints (e.g., line limits)

The need for generator reserves

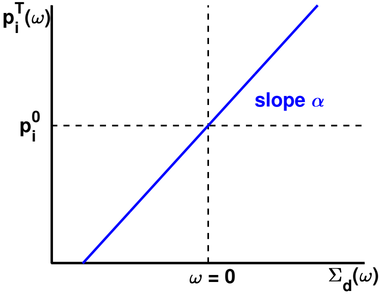
- High wind power penetration + Intermittency of wind energy
  $\implies$ need to balance power using reserves
- Reserve power can be provided by both conventional and wind power sources
- Existing models of reserve activation are inadequate
- Bottom line: considering the effect of uncertainties can enable safe *and* economically efficient operation of the grid

Managing reserve saturation

- Activate reserves to counteract imbalance in power due to load and wind uncertainties ω

$$\underbrace{p_i^{\mathrm{T}}(\omega)}_{\substack{\text{target}\\\text{generation}}} = \underbrace{p_i^0}_{\substack{\text{nominal}\\\text{generation}}} + \underbrace{\alpha_i \Sigma_d(\omega)}_{\substack{\text{reserve}\\\text{generation}}}$$

$\alpha_i =$ participation factor; $\sum \alpha_i = 1$
$\Sigma_d(\omega) =$ power imbalance



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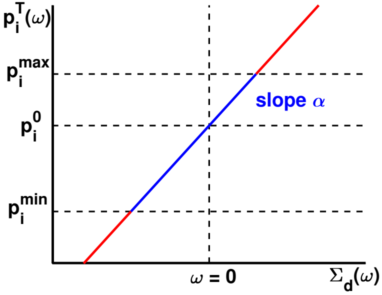

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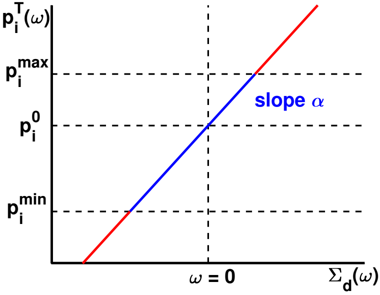

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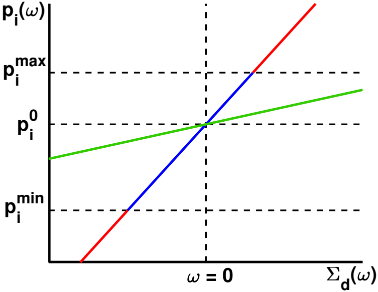

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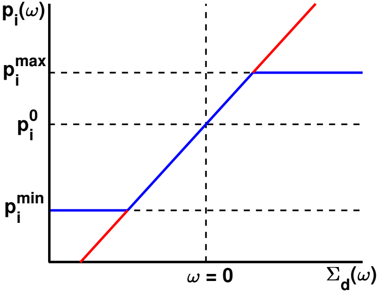


- How to avoid asking a generator for more than it can produce?
  - Reduce the slope α_i
  - Model reserve saturation + redistribute additional reserves

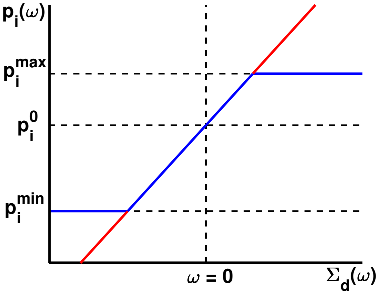
Modeling reserve saturation

Reserve saturation: Generators provide reserves only until they hit limits

- Activate reserves in response to load and wind uncertainties ω

$$\underbrace{p_i^{\mathrm{T}}(\omega)}_{\substack{\text{target}\\ \text{generation}}} = p_i^0 + \alpha_i \Sigma_d(\omega) + \alpha_i s(\omega)$$

$$\alpha_i s(\omega) = \text{"shortfall" in generation due to reserve saturation}$$



- *Actual generation* $p_i(\omega)$ decided as

$$p_i(\omega) := \begin{cases} p_i^{\min}(\omega), & \text{if } p_i^{\mathrm{T}}(\omega) < p_i^{\min}(\omega) \\ p_i^{\mathrm{T}}(\omega), & \text{if } p_i^{\min}(\omega) \leq p_i^{\mathrm{T}}(\omega) \leq p_i^{\max}(\omega) \\ p_i^{\max}(\omega), & \text{if } p_i^{\mathrm{T}}(\omega) > p_i^{\max}(\omega) \end{cases}$$

Stochastic DC-OPF with reserve saturation

Two-stage stochastic DC-OPF model

- First stage: Determine nominal/scheduled generation and participation factors, and procure reserves
- Second stage: Determine actual generation levels and other power flow variables, penalize line flow violations, and minimize exceedance of procured reserves
- Use reserve saturation in the second stage to determine actual generation levels

First-stage model

Decision variables: for each generator $i \in \mathcal{G}$

$p_i^0 =$ scheduled generation, $\quad \alpha_i =$ participation factor

$r_i^+ =$ procured up-reserves, $\quad r_i^- =$ procured down-reserves

$$\min_{\substack{p^0,\alpha \\ r^+,r^-}} \sum_{i \in \mathcal{G}} \overbrace{\Big(f_{1,i}(p_i^0) + f_{2,i}(r_i^+) + f_{3,i}(r_i^-) \Big)}^{\text{cost of scheduled generation and reserves}} + \overbrace{Q(p^0, r^+, r^-, \alpha)}^{\text{expected second-stage cost}}$$

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$$\left. \begin{array}{l} p_i^0 + r_i^+ \leq p_i^{\max}, \ p_i^0 - r_i^- \geq p_i^{\min}, \quad \forall i \in \mathcal{R}, \\ 0 \leq r^+ \leq r^{+,\max}, \ 0 \leq r^- \leq r^{-,\max}, \end{array} \right\} \begin{array}{c}\text{Scheduled generation} \\ \text{and reserves within} \\ \text{generator bounds}\end{array}$$

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$$\left. \alpha \geq 0, \ \sum_{i \in \mathcal{G}} \alpha_i = 1, \ \alpha_i \geq \alpha^{\min}, \ \forall i \in \mathcal{G}^{\mathrm{res}} \right\} \begin{array}{c}\text{A subset of generators are} \\ \text{required to provide reserves}\end{array}$$

Second-stage model

Expected second-stage cost $Q(p^0, r^+, r^-, \alpha) = \mathbb{E}_\omega[q(p^0, r^+, r^-, \alpha, \omega)]$, where $q(p^0, r^+, r^-, \alpha, \omega)$ is the optimal value of

$$\min_{\substack{p(\omega), p^\mathsf{T}(\omega) \\ s(\omega), \theta(\omega)}} \sum_{i \in \mathcal{G}} \overbrace{\Big(q_{1,i}(p_i(\omega) - (p_i^0 + r_i^+)) + q_{2,i}(p_i(\omega) - (p_i^0 - r_i^-))\Big)}^{\text{penalty for exceeding procured reserves}} +$$

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$$\text{reserve saturation model} \left\{ p_i(\omega) = \begin{cases} p_i^{\min}(\omega), & \text{if } p_i^{\mathsf{T}}(\omega) < p_i^{\min}(\omega) \\ p_i^{\mathsf{T}}(\omega), & \text{if } p_i^{\min}(\omega) \leq p_i^{\mathsf{T}}(\omega) \leq p_i^{\max}(\omega) \\ p_i^{\max}(\omega), & \text{if } p_i^{\mathsf{T}}(\omega) > p_i^{\max}(\omega) \end{cases}, \;\; \forall i \in \mathcal{G}, \right.$$

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$$\text{DC power flow equations} \left\{ \sum_{j:(i,j) \in \mathcal{E}} \beta_{ij}[\theta_i(\omega) - \theta_j(\omega)] = p_i(\omega) - d_i - \tilde{d}_i(\omega), \;\; \forall i \in \mathcal{V} \right.$$

Tailored decomposition method

- Second-stage problem: smooth convex objective function, but nonsmooth, nonconvex constraints (due to reserve saturation)

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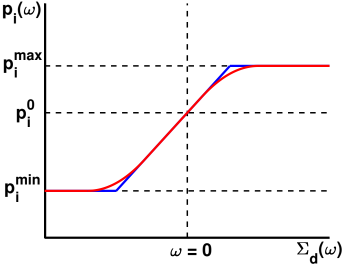
  - Construct smooth approximation of reserve saturation equation



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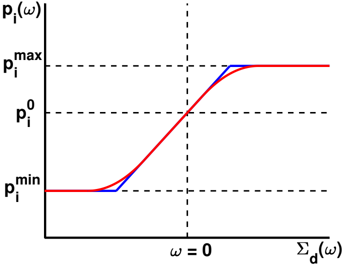

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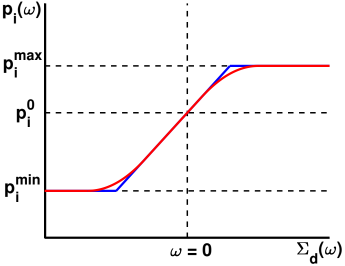

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  - Compute second-stage solution by binary search + linear solve
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- Use projected stochastic gradient method to compute stationary solution to approx. (Davis and Drusvyatskiy, 2018)

Computational experiments

Modify IEEE 6-bus and 118-bus instances to include wind power plants

6-bus case: 10% load uncertainty; 118-bus case: 5% load uncertainty

Both cases: 10% wind uncertainty (+ correlations across wind plants)

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Compare

- Our approach: Smooth approximation of reserve saturation
- Generator penalty approach: No reserve saturation, but penalize generator bounds violations in second stage
- Chance-constrained approach: No reserve saturation, but limit individual probabilities of generator bounds violations

Second-stage objective function is *nearly* the same for all approaches

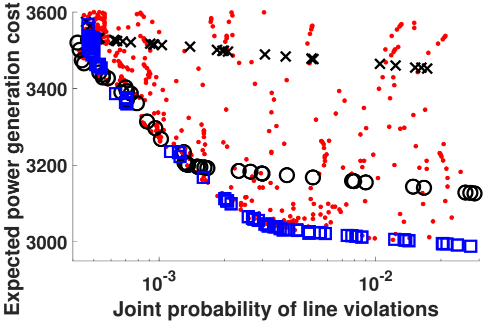
- Extra generator bounds violation penalty for the GP approach

Evaluate out-of-sample performance of all three approaches

- Include reserve saturation for all approaches in this evaluation

IEEE 6-bus system with one wind power plant

Case 1: Wind generator can provide reserves

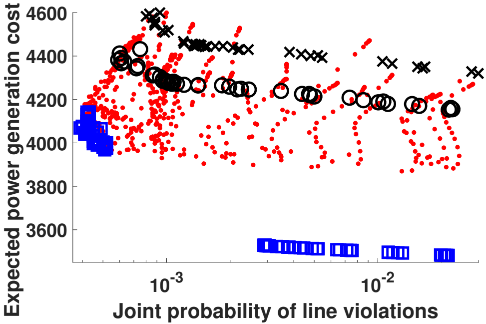


□: our approach. ●: generator penalty. ○ and ×: chance constraints

- Our approach outperforms chance-constrained method
- Our approach exhibits similar performance as generator penalty method, but with less tuning

IEEE 6-bus system with one wind power plant

Case 2: Wind generator does NOT provide reserves



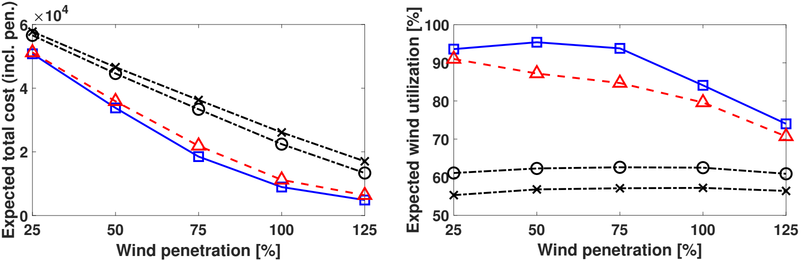
□: our approach. ●: generator penalty. ○ and ×: chance constraints

- Our approach outperforms both chance-constrained and generator penalty methods
- Gap in efficient frontier from our approach because we use a penalty approach and the efficient frontier is nonconvex

IEEE 118-bus system with 25 wind power plants

- Consider different wind power penetration levels
- Plot lowest cost solution with line flow violations $\leq 0.5\%$
- Our approach yields solutions with

Lower total cost and Higher wind utilization



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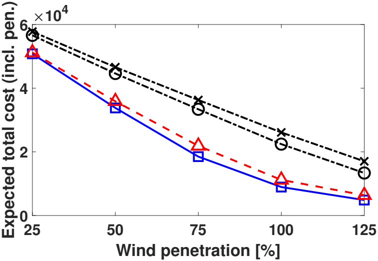

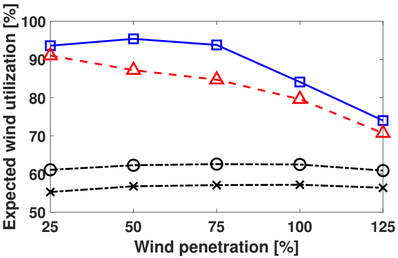

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Reasonable computational effort for all methods

- Smooth approximation: 7 min; other two methods: 1.5 min

Concluding remarks

- Modeling reserve saturation enables effective use of wind power resources
- Resulting nonsmooth, nonconvex two-stage stochastic program is solved using a tailored decomposition algorithm
- Our approach outperforms formulations that use chance constraints and generator bounds violation penalties

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